Physics 448 **HOMEWORK SET 4** **Fall 2008**

Due Wednesday October 1

18) The standard deviation, $\sigma_{\rm x}$, of a distribution is defined as $\sigma_{\rm x}^2 = \langle (\rm x - \langle x \rangle)^2 \rangle$. Show that this is equivalent to $\sigma_{\rm x}^2 = \langle \rm x^2 \rangle - \langle x \rangle^2$.

19) In this problem we use a weighting function $A(k) = \rm C$ (a constant) for $k_0 - \Delta k/2 < k < k_0 + \Delta k/2$, and $A(k) = 0$ elsewhere.

    (a) Find the normalization constant C that gives $\int_{-\infty}^{\infty} |A(k)|^2 dk = 1$.

    (b) Find the $t = 0$ wave function $\Psi(x, 0)$, and show that this wave function is properly normalized.

20) (a) Find the wave function $\psi(x, t = 0)$ corresponding to

$$A(k) = \frac{N}{(k - k_0)^2 + \alpha^2}.$$

    (b) Normalize the wave function and calculate $\sigma_{\rm k}$ and $\sigma_{\rm x}$. Compare your answer with the uncertainty principle rule $\sigma_{\rm k}\,\sigma_{\rm x} \geq \frac{1}{2}$.

21) In class we obtained a Gaussian wave packet of the form

$$\Psi(x, t) = \frac{C}{\sqrt{2}} \left[\frac{1}{a + i\gamma t}\right]^{1/2} e^{i(k_0 x - \omega_0 t)}\, e^{-(x-\beta t)^2/4(a+i\gamma t)}.$$

    (a) Find the probability distribution $P(x) = |\psi(x, t)|^2$.

    (b) Show that the integral of $P(x)$ over all x does not depend on time.

22) The energy of a particle of mass m subject to a harmonic oscillator potential is

$$E = T + V = \frac{p^2}{2m} + \frac{1}{2}kx^2.$$

Since $T \propto p^2$ and $V \propto x^2$, E can never be negative. But with the uncertainty principle one can show that small positive values of E are also ruled out. Find the minimum value of E consistent with the uncertainty principle $\delta p \cdot \delta x \geq \hbar/2$. [Hints: Write $\langle E \rangle$ in terms of $\langle p^2 \rangle$ and $\langle x^2 \rangle$ and then use the result of Problem 18 to express $\langle E \rangle$ in terms of the uncertainties.]