Physics 448 **HOMEWORK SET 5** **Fall 2008**

Due Wednesday October 8

23) Gasiorowicz problem 2.2.

24) The dispersion relation for gravitational waves in water of depth h is

$$\omega^2 = gk(1 - e^{-2kh})/(1 + e^{-2kh}).$$

Calculate the phase and group velocities for deep water ($h >> \lambda$) and shallow water ($h << \lambda$). In which limit are the waves nondispersive?

25) Gasiorowicz problem 2.6.

26) Suppose that a particle of mass m has a wave function

$$\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t}.$$

(a) Normalize Ψ.

(b) Find the expectation values of x and x^2.

(c) Sketch a graph of $|\Psi|^2$ and mark the points $\langle x\rangle + \sigma$ and $\langle x\rangle - \sigma$. Calculate the probability that the particle would be found outside this range.

27) (a) Find the probability current for a wave function of the form $\psi_1(x) = Ce^{\alpha x} + De^{-\alpha x}$ where C and D may in general be complex.

(b) Show that if we match a wave function of the form $\psi_2(x) = Ae^{ikx}$ to the function ψ_1 given above at some point $x = a$, the probability current will be the same on both sides of the matching point.

28) (a) Set up the matching equations for a particle of mass m and energy E incident from the left on a rectangular well

$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ -V_0 & \text{for } 0 < x < a \\ 0 & \text{for } x > a\ . \end{cases}$$

where

(b) Solve the matching equations for the ratio of the amplitude of the transmitted wave to the amplitude of the incident wave, and then find the transmission probability.

(c) If the incident particle is an electron of kinetic energy 5 eV and the potential is 3 eV deep, what is the minimum non-zero value of a for which there is perfect transmission ($T = 1$).