**Physics 448** **HOMEWORK SET 7** **Fall 2008**

## Due Wednesday November 5

35) Suppose we have a free particle of mass $m$ described at $t = 0$ by a normalized time-dependent wave function

$$\Psi(x, t = 0) \;=\; \left(\frac{1}{a\sqrt{\pi}}\right)^{\frac{1}{2}} e^{-x^2/2a^2}\, e^{ip_0x/\hbar}.$$

The corresponding normalized momentum-space wave function is then

$$\Phi(p, t) = \left(\frac{a}{\hbar\sqrt{\pi}}\right)^{\frac{1}{2}} e^{-a^2(p-p_0)^2/2\hbar^2}\, e^{-iEt/\hbar},$$

where $E = p^2/2m$.

(a) Find $\langle x\rangle$ and $\langle p\rangle$ at $t = 0$ using the *coordinate*-space wave function and operators.

(b) Using the *momentum*-space wave function and operators, find $\langle x\rangle$ and $\langle p\rangle$ for all $t$.

36) In this problem $\psi_0(x)$, $\psi_1(x)$ and $\psi_2(x)$ are the first three energy eigenfunctions for the harmonic oscillator.

(a) Using equations given in class, write out each wave functions in full detail including the normalization constant.

(b) Show by explicit calculation that $\psi_0(x)$ and $\psi_2(x)$ are orthogonal.

37) (a) Use the recursion formula

$$a_{m+2} = \frac{2m + 1 - \epsilon}{(m+1)(m+2)}\, a_m,$$

to generate the function $H_5(y)$. Fix the overall normalization by setting $a_5$ to the value $a_n = 2^n$.

(b) Show that you get the same result from the "generating function",

$$H_n(y) = (-1)^n\, e^{y^2} \frac{d^n}{dy^n}\, e^{-y^2}.$$

(c) Starting from $H_0 = 1$, find $H_5(y)$ by repeated application of the "recursion relation,"

$$H_{n+1}(y) = 2\,y\,H_n(y) - 2\,n\,H_{n-1}(y).$$

38) Find the expectation values of the kinetic energy $T = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$ and the potential energy $V = \frac{1}{2}kx^2$ for a particle in an arbitrary harmonic oscillator state $\psi_{\rm n}$. Do the calculations using the formulas

$$x\psi_n = \frac{1}{\sqrt{2a}}\left[\sqrt{n+1}\,\psi_{n+1} + \sqrt{n}\,\psi_{n-1}\right] \quad \text{and} \quad \frac{d}{dx}\psi_n = \sqrt{\frac{a}{2}}\left[-\sqrt{n+1}\,\psi_{n+1} + \sqrt{n}\,\psi_{n-1}\right]$$

and show that $\langle T\rangle + \langle V\rangle = E_n$.

39) A particle of mass $m$ is confined in a harmonic oscillator well. Suppose that at time $t=0$ the particle is in a quantum state described by the "wave packet"

$$\Psi(x, t=0) = \sqrt{\frac{2}{3}} \left[\frac{a}{\pi}\right]^{\frac{1}{4}} \left[1 + \sqrt{a}x\right] e^{-ax^2/2}$$

where $a = (\sqrt{km}/\hbar)$.

(a) Find the full time dependent wave function by expanding $\Psi$ in terms of energy eigenfunctions $\psi_0$ and $\psi_1$.

(b) Find $\langle x \rangle$ as a function of time for this wave function.

40) In the $H_2$ molecule the potential energy between the two atoms can be approximated by

$$V(x) = A_0 \left[\left(\frac{a}{x}\right)^4 - \left(\frac{a}{x}\right)^2\right]$$

where $A_0 = 20\,\mathrm{eV}$ and $a = 0.05\,\mathrm{nm}$.

(a) Make a sketch of $V(x)$.

(b) Find the equilibrium separation $x_0$.

(c) For $x$ close to $x_0$ we can write

$$V(x) = V_0 + \left[\frac{dV}{dx}\right]_{x_0} (x - x_0) + \frac{1}{2}\left[\frac{d^2V}{dx^2}\right]_{x_0} (x - x_0)^2 + \ . \ . \ .$$

and since the slope of $V(x)$ is zero at the equilibrium point we can approximate $V(x)$ by

$$V(x) \simeq V_0 + \frac{1}{2}k(x - x_0)^2.$$

Use this approximation to predict the energy spacing (in eV) between the vibrational states of the molecule. For m use the reduced mass $\mu$.