Physics 448 **HOMEWORK SET 8** **Fall 2008**

Due Wednesday November 12

41) In class we showed that in the harmonic oscillator

$$A\psi_n = \sqrt{n}\,\psi_{n-1} \qquad \text{and} \qquad A^\dagger\psi_n = \sqrt{n+1}\,\psi_{n+1}$$

where

$$A = \sqrt{\frac{a}{2}}\,x + \frac{i}{\sqrt{2a}}\frac{p}{\hbar} \qquad \text{and} \qquad A^\dagger = \sqrt{\frac{a}{2}}\,x + \frac{i}{\sqrt{2a}}\frac{p}{\hbar},$$

and where $a = \sqrt{km}/\hbar$. Use these results to derive the following useful formulas:

$$x\,\psi_n(x) = \frac{1}{\sqrt{2a}}\left[\sqrt{n}\,\psi_{n-1} + \sqrt{n+1}\,\psi_{n+1}\right]$$

and

$$\frac{d}{dx}\psi_n(x) = \sqrt{\frac{a}{2}}\left[\sqrt{n}\,\psi_{n-1} - \sqrt{n+1}\,\psi_{n+1}\right]$$

42) Determine whether the following operators are linear?

(a) $\mathrm{Q}_1\psi = \mathrm{x}^2\psi$

(b) $\mathrm{Q}_2\psi = \mathrm{x}\frac{\partial}{\partial \mathrm{x}}\psi$

(c) $\mathrm{Q}_3\psi = \psi^*$

(d) $\mathrm{Q}_4\psi = \int_{-\infty}^{\mathrm{x}} \mathrm{x}'\psi(\mathrm{x}')\mathrm{dx}'$

(e) $\mathrm{Q}_5\psi = \ell n\psi$

43) A particle in a harmonic oscillator well is described at time $t = 0$ by the wave function

$$\Psi(x,0) = N\left[(3+2i)\psi_0(x) - 3\psi_1(x) + 5i\psi_2(x)\right]$$

where the ψ's are the energy eigenfunctions.

(a) Find the normalization constant N.

(b) Find the probability that a measurement of E would give: $\frac{1}{2}\hbar\omega$; $\frac{3}{2}\hbar\omega$; $\frac{5}{2}\hbar\omega$.

(c) Determine the expectation value of E.

44) For a particle in an infinite square well extending from 0 to L, the energy eigenfunctions are given by $\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi}{L}x$ with eigenvalues $E_n = n^2\pi^2\hbar^2/2mL^2$. Suppose we start a particle in the well with an initial (normalized) wave function

$$\Psi(x,0) = \begin{cases} -\frac{1}{\sqrt{L}} & \text{for } 0 < x < \frac{L}{2} \\ +\frac{1}{\sqrt{L}} & \text{for } \frac{L}{2} < x < L \end{cases}$$

Find the probability that a measurement of the energy would give: E_1; E_2; E_3; E_4. [Hint: You can save a lot of time if you sketch the functions before trying to do any integrals.]

45) Show that if A and B are Hermitian, $(A+B)^2$ is also Hermitian. Assume that A and B do not commute.

46) Show that if A and B are Hermitian, $i[A,B]$ is also Hermitian.