Physics 448 **HOMEWORK SET 9** **Fall 2008**

Due Wednesday November 19

47) Show that the functions $\psi_1(x) = e^{\alpha x}$ and $\psi_2(x) = e^{-\alpha x}$ are eigenfunctions of the operator $Q = \frac{d^2}{dx^2}$ with the same eigenvalue. Construct two linear combinations of ψ_1 and ψ_2 that are orthogonal to each other and normalized on the interval $(-L, L)$.

48) Consider the matrix operator

$$T = \begin{pmatrix} 1 & 1-i \\ 1+i & 0 \end{pmatrix}.$$

(a) Find the expectation value of T in an arbitrary state $\psi = \begin{pmatrix} a \\ b \end{pmatrix}$ and show that the expectation value of T is real for all ψ.

(b) Find the eigenvalues of T.

(c) Normalize the eigenvectors and show that they are orthogonal.

49) Your job in this problem is to understand how to write the operator x in matrix form. For the basis functions, ψ_n, we will use the harmonic oscillator energy eigenfunctions. Of course the matrix is infinitely large, since there are an infinite number of basis states, so I will be satisfied if you can find the upper left 5 by 5 corner. The problem is actually very simple if you just use the results from Problem 41. Is your matrix Hermitian ($T_{ij} = T^*_{ji}$)?

50) Suppose that the energy operator H for a 3-state system can be written in the form

$$H = \begin{pmatrix} a & 0 & a \\ 0 & b & 0 \\ a & 0 & a \end{pmatrix}.$$

(a) Find the energy eigenvalues.

(b) Suppose the system starts out at $t = 0$ in the state $\psi = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Find $\psi(t)$ for $t > 0$.

51) Prove the following commutator identities:

(a) $[\mathrm{A}, \mathrm{B}] = -[\mathrm{B}, \mathrm{A}]$

(b) $[\mathrm{AB}, \mathrm{C}] = \mathrm{A}[\mathrm{B}, \mathrm{C}] + [\mathrm{A}, \mathrm{C}]\mathrm{B}$

(c) $[\mathrm{A}, \mathrm{BC}] = \mathrm{B}[\mathrm{A}, \mathrm{C}] + [\mathrm{A}, \mathrm{B}]\mathrm{C}$

(d) $[\mathrm{A}, [\mathrm{B}, \mathrm{C}]] + [\mathrm{B}, [\mathrm{C}, \mathrm{A}]] + [\mathrm{C}, [\mathrm{A}, \mathrm{B}]] = 0$

52) Let A and B be any two Hermitian operators. Show that the product AB is Hermitian IF and ONLY IF [A,B] = 0. COMMENTS: This is an if and only if proof so you need to demonstrate that *i*) if [A,B] = 0 then AB is Hermitian and *ii*) if AB is Hermitian then [A,B] = 0. It may be helpful to postulate the existence of a complete orthonormal set of states $\{\psi_n\}$.