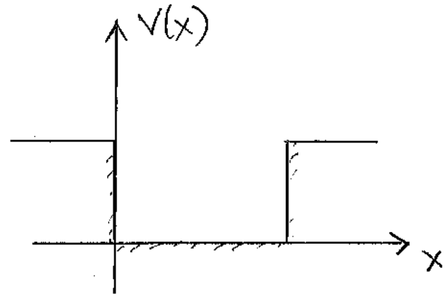
Physics 448 **EXAM 1** **October 21, 2005**

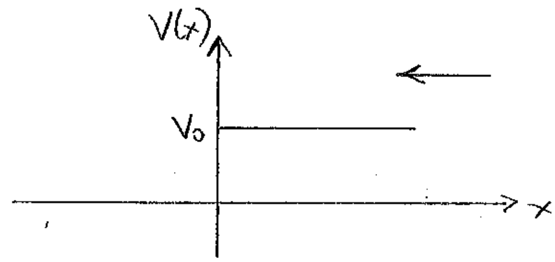
Each problem is worth $33\frac{1}{3}$ points.



1) (a) Use the Wilson-Sommerfeld quantization rule ($\oint p\,dq = nh$) to find the allowed energies of a particle of mass m confined in a square-well potential of width L.

   (b) Find the wavelength of the photons emitted in the $n=2 \to n=1$ transition assuming the confined particle is an electron and that the well has a width $L = 0.5\,\text{nm}$.

2) A particle of mass m is incident <u>from the right</u> on a potential step as shown in the drawing. Assume that $E > V_0$. The general form of the wave function is then



$$\psi(x) = \begin{cases} \_\_\_e^{i\alpha x} + \_\_\_e^{-i\alpha x} & \text{for } x > 0 \\ \_\_\_e^{i\beta x} + \_\_\_e^{-i\beta x} & \text{for } x < 0 \end{cases}$$

   (a) Find the appropriate expressions for α and β in terms of E and V_0.

   (b) Which term should be set to zero for particles incident from the right?

   (c) Fill in the remaining blanks with coefficients A, B and C, using A for the incident wave, B for the reflected wave and C for the transmitted wave.

   (d) Match the wave functions and solve for C in terms of A.

   (e) Find the transmission probability.

3) A free electron has kinetic energy $10\,\text{eV}$. At time $t = 0$ the electon is at $x = 0$ and is described by a Gaussian wave packet of width $\sigma_x = 10^{-4}\,\text{m} = 10^5\,\text{nm}$. We measure the arrival time of the electron at a point $x = d$.

   (a) Calculate the velocity and the expected arrival time for $d = 10\,\text{m}$.

   (b) Find the uncertainty (σ) in the arrival time for $d = 10\,\text{m}$.

   (c) Find the uncertainty (σ) in the arrival time for $d = 10\,\text{km}$.

<u>Hints</u>: Gaussian wave packets give a probability distribution, $P(x,t)$, that can be written in the form

$$P(x,t) = \frac{1}{\sqrt{2\pi}}\left[\frac{1}{\sigma_0^2 + (\gamma t/\sigma_0)^2}\right]^{\frac{1}{2}} e^{-(x-\beta t)^2/2[\sigma_0^2 + (\gamma t/\sigma_0)^2]}$$

where $\beta = \frac{d\omega}{dk}$, $\gamma = \frac{1}{2}\frac{d^2\omega}{dk^2}$ and σ_0 is the packet width at $t=0$. Some ways of doing the problem would use this formula.