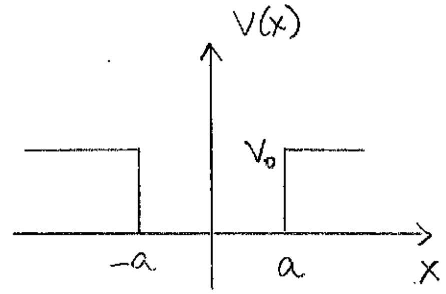
Physics 448 **EXAM 2** **November 30, 2005**

Each problem is worth $33\frac{1}{3}$ points.

1) An electron is confined in a bound state of a finite square potential as shown in the drawing.



   (a) Write down the general expressions for the wave function $\psi(x)$ in the regions $x < -a$, $-a < x < a$ and $x > a$. Eliminate terms that are disallowed for one reason or another.

   (b) Given that H commutes with the parity operator $[P\psi(x) = \psi(-x)]$, explain in detail what you can conclude about the coefficients (A, B, C *etc.*) that appear in the wave functions you wrote down in part (a).

   (c) Using results from (b) that are applicable for the **second excited state**, match the wave functions at $x = a$ and find the transcendental equation for the energy of the state.

   (d) How large must V_0 be in order for the second excited state to actually be bound? Show work and/or explain your thinking. (You will get only minimal credit for "finding the right equation" on your formula card.)

2) This is a harmonic oscillator problem, $H = p^2/2m + \frac{1}{2}kx^2$.

   (a) Work out the commutation relation $[H, Q]$, where $Q = x + ip/\sqrt{km}$. You should be able to express the answer in terms of Q.

   (b) Suppose ψ is an eigenstate of H with eigenvalue E. Show that $Q\psi$ is also an eigenfunction of H. What is the eigenvalue in this case?

3) Consider a two-state system in which the energy operator can be written in the form

$$H = \begin{pmatrix} 0 & iA \\ -iA & 0 \end{pmatrix},$$

where A is real.

   (a) Find the energy eigenvalues and the corresponding eigenvectors, ψ_1 and ψ_2.

   (b) Normalize ψ_1 and ψ_2.

   (c) Suppose the system starts out at $t = 0$ in a state $\psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Expand ψ in terms of the eigenstates and find $\psi(t)$. If possible, simplify your result by looking for combinations that reduce to $\cos\omega t$ and $\sin\omega t$ where $\omega = A/\hbar$.