Physics 448 **FINAL EXAM** **December 21, 2000**

1) [20 points]
(a) Determine whether the components of the angular momentum (L_x, L_y and L_z) commute with x^2+y^2.
(b) Suppose we have a particle moving under the influence of a potential V that depends only on $\rho = x^2+y^2$. Which of the following quantities

$$L_x \quad L_y \quad L_z \quad x \quad y \quad z \quad p_x \quad p_y \quad p_z$$

will be constants of the motion? [All answers must include some sort of work or explanation.]

2) [10 points]
An operator is said to be Hermitian if $\langle \psi \,|\, Q\psi \rangle = \langle Q\psi \,|\, \psi \rangle$ for all "well behaved" (i.e. physically acceptable) wave functions ψ. Using this definition, show that $p_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$ is Hermitian.

3) [20 points]
The wave functions for the hydrogen atom (taking $Z = 1$) have the form

$$\psi(r,\theta,\phi) = R(r) Y_l^m(\theta,\phi) \qquad \text{where} \qquad R(r) = N(\alpha r)^l \sum_k a_k (\alpha r)^k \, e^{-\alpha r}$$

with the recursion formula

$$a_{k+1} = \frac{(k+l+1)-\sigma}{(k+1)(k+2l+2)} a_k$$

where

$$\alpha = \left[\frac{8\mu|E|}{\hbar^2}\right]^{\frac{1}{2}} \qquad \text{and} \qquad \sigma = \left(\frac{e^2}{4\pi\epsilon_0}\right)\frac{1}{\hbar c}\left[\frac{\mu c^2}{2|E|}\right]^{\frac{1}{2}}$$

(a) What must happen in order for ψ to remain finite in the limit $r \to \infty$.
(b) Show how the requirement that ψ be finite leads to energy quantization, and find the formula for the energy.
(c) For what values of l does one obtain eigenfunctions with energy $E = E_0/9$ where E_0 is the ground state energy? Explain.
(d) How many degenerate states are there for this energy? List the quantum numbers of each of these states.

4) [20 points]
The eigenfunctions of L^2 and L_z corresponding to $l = 1$ are given by

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\frac{x+iy}{r} \qquad Y_1^0 = \sqrt{\frac{3}{4\pi}}\frac{z}{r} \qquad Y_1^{-1} = \sqrt{\frac{3}{8\pi}}\frac{x-iy}{r}$$

If a particle with $l = 1$ is known to have $L_x = +\hbar$, find the probabilities that subsequent measurements of L_z yield the values $L_z = +\hbar$, $L_z = 0$, and $L_z = -\hbar$.

5) [15 points]
Find the expectation value of the potential energy for a hydrogen atom in the state

$$\psi_{2,1,0} = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{\frac{3}{2}} \frac{r}{a_0} e^{-r/2a_0} \cos\theta.$$

where $a_0 = (\hbar^2/\mu)(4\pi\epsilon_0/e^2)$. [The following integral will be useful: $\int_0^\infty x^n e^{-x} dx = n!$]