12) In general the Bohr energies are $E_n = -\frac{mc^2}{2}\left(\frac{e^2}{4\pi\epsilon_0}\cdot\frac{1}{\hbar c}\right)^2 \frac{1}{n^2}$
so for $n=3 \rightarrow n=2$

$$h\nu = \frac{hc}{\lambda} = \frac{mc^2}{2}\left(\frac{e^2}{4\pi\epsilon_0}\cdot\frac{1}{\hbar c}\right)^2\left(\frac{1}{4}-\frac{1}{9}\right) \qquad m = \text{reduced mass}$$

$$\frac{1}{\lambda} = \frac{mc^2}{4\pi}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\left(\frac{1}{\hbar c}\right)^3\left(\frac{1}{4}-\frac{1}{9}\right)$$

The wavelength differences will be small so we need to calculate λ to many decimal places. First the reduced mass

Hydrogen: $mc^2 = \frac{(m_ec^2)(m_pc^2)}{m_ec^2+m_pc^2} = \frac{(0.511)(938.3)}{0.511+938.3} = 0.5107219$ MeV.

Similarly

Deuterium: $mc^2 = 0.5108608$ MeV

Tritium $mc^2 = 0.5109071$ MeV.

$$\frac{1}{\lambda} = \frac{mc^2}{4\pi}(1.44\,\text{eV}\cdot\text{nm})^2\left(\frac{1}{197.3\,\text{eV}\cdot\text{nm}}\right)^3\left(\frac{1}{4}-\frac{1}{9}\right)$$

gives

$\lambda_H = 656.4783$ nm

$\lambda_D = 656.2997$ nm

$\lambda_T = 656.2403$ nm

$\Delta\lambda$ (H-D) =	0.179 nm
$\Delta\lambda$ (D-T) =	0.059 nm

13) If we drop the mass from a height $z=H$ it will fall and bounce at $z=0$. Assume energy is conserved to get periodic motion (bouncing). Use coordinate z. Then

$$T = \tfrac{1}{2}m\dot{z}^2 \qquad \mathcal{L} = T - V = \tfrac{1}{2}m\dot{z}^2 - mgz$$

$$p = \frac{\partial\mathcal{L}}{\partial\dot{z}} = m\dot{z} = \text{the usual thing.}$$

Energy is conserved so

$$E = p^2/2m + mgz = \text{constant} = mgH$$

$$p = \pm\left[2m^2 g(H-z)\right]^{\frac{1}{2}}$$

$$\oint p\,dq = \int_0^H \left[2m^2 g(H-z)\right]^{\frac{1}{2}} dz + \int_H^0 (-)\left[2m^2 g(H-z)\right]^{\frac{1}{2}} dz$$

$$= 2m\sqrt{2g}\int_0^H (H-z)^{\frac{1}{2}} dz = 2m\sqrt{2g}\left(-\tfrac{2}{3}\right)(H-z)^{3/2}\Big|_0^H$$

$$= 2m\sqrt{2g}\left(+\tfrac{2}{3}\right)H^{3/2} = \tfrac{4}{3}m\sqrt{2g}\,H^{\frac{3}{2}} = nh$$

$$H = \left(\tfrac{3}{4}nh/\sqrt{2g}\,m\right)^{2/3}$$

$$E_n = mgH = mg\left(\tfrac{3}{4}nh/\sqrt{2g}\,m\right)^{2/3} = mg\left[\frac{9n^2h^2}{32m^2g}\right]^{\frac{1}{3}}$$

$$\boxed{E_n = \left[\tfrac{9}{32}mg^2n^2h^2\right]^{\frac{1}{3}}}$$

14) As the body rotates the periodic coordinate is θ, and the corresponding velocity is $\dot{\theta} \equiv \omega$. The rotational KE is

$$T = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}I\dot{\theta}^2$$

The potential energy is zero, so

$$p_\theta = \frac{\partial T}{\partial\dot{\theta}} = I\dot{\theta} = L = \text{constant}$$

$$\oint p\,dq = \int_0^{2\pi} I\dot{\theta}\,d\theta = I\dot{\theta}\int_0^{2\pi} d\theta = 2\pi I\dot{\theta} = nh$$

$$L = I\dot{\theta} = n\hbar \qquad T = E = \frac{L^2}{2I} \Rightarrow \boxed{E_n = \frac{n^2\hbar^2}{2I}}$$

15)(a) $E = -\frac{1}{2}\alpha^2 mc^2\left[1 + \frac{\alpha^2}{n}\left(\frac{1}{k} - \frac{3}{4n}\right)\right]\frac{1}{n^2}$

So

$E(k=2) - E(k=1)$

$$= -\frac{1}{2}(\alpha^2 mc^2)\frac{1}{n^2}\left[\frac{\alpha^2}{n}\left(\frac{1}{2} - \frac{1}{1}\right)\right] = \frac{1}{4}\frac{\alpha^4}{n^3}mc^2$$

$$= \frac{1}{4}\left(\frac{1.44\,\text{eV}\cdot\text{nm}}{197.3\,\text{eV}\cdot\text{nm}}\right)^4 \frac{1}{8}(5.11\times10^5\,\text{eV}) = \boxed{4.5\times10^{-5}\,\text{eV}}$$

(b) $\frac{hc}{\lambda} = E(n=3) - E(n=2) \Rightarrow \lambda = \frac{hc}{E_3 - E_2}$

So how much does λ change if we change E_2 by a small amount 4.5×10^{-5} eV.

$$\Delta\lambda = \frac{d\lambda}{dE_2}\Delta E_2 = (-)\frac{hc}{(E_3-E_2)^2}(-1)\cdot\Delta E_2$$

$$= \left(\frac{hc}{E_3-E_2}\right)\frac{\Delta E_2}{E_3-E_2} = \lambda\frac{\Delta E_2}{E_3-E_2} \qquad \lambda \sim 656\,\text{nm}$$

$$E_3 \doteq -\frac{13.6\,\text{eV}}{9} = -1.51\,\text{eV} \qquad E_2 \doteq -\frac{13.6\,\text{eV}}{4} = -3.40\,\text{eV}$$

$$\Delta\lambda \doteq (656\,\text{nm})\frac{4.5\times10^{-5}\,\text{eV}}{(3.40-1.51)\,\text{eV}} = \boxed{0.0156\,\text{nm}}$$

16)(a) From class we had

$$\frac{1}{E'} = \frac{1}{E} + \frac{1}{m_e c^2}(1-\cos\theta) = \frac{1}{300\,\text{keV}} + \frac{1}{511\,\text{keV}}(1-\cos 45^\circ)$$

$\Rightarrow$

$$\boxed{E' = 256\,\text{keV}}$$

So the electron gains 44 keV of kinetic energy.

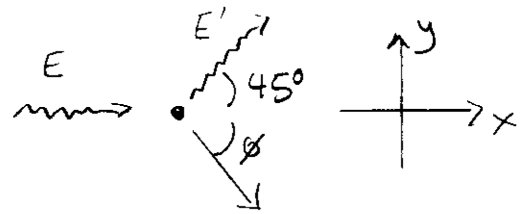
(b) $E_{TOT} = T + mc^2 = (44 + 511)\,\text{keV} = 555\,\text{keV}$

$$E^2 = (pc)^2 + (mc^2)^2$$

so

$$pc = \left[E^2 - (mc^2)^2\right]^{\frac{1}{2}} \qquad \boxed{p = 216.6 \text{ keV}/c}$$

To find the direction we can use the fact that momentum is conserved



$$p_e \sin\phi = p' \sin 45^\circ$$

$$\sin\phi = \frac{p'c}{p_e c}\sin 45^\circ = \frac{256 \text{ keV}}{216.6 \text{ keV}}\sin 45^\circ \qquad \boxed{\phi = 56.7^\circ}$$

17) (a) $T \ll mc^2$ so the electron is non-relativistic.

$$T = \frac{1}{2}mv^2 = \frac{p^2}{2m} \qquad pc = \left[2Tmc^2\right]^{\frac{1}{2}} = \left[2(10\text{eV})(5.11\times10^5\text{eV})\right]^{\frac{1}{2}}$$

$$= 3197 \text{ eV}.$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1240 \text{ eV.nm}}{3197 \text{ eV}} \qquad \boxed{\lambda = 0.388 \text{ nm}}$$

(b) These electrons are relativistic $E_{TOT} = T + mc^2 = 50 \text{ MeV} + 0.511 \text{ MeV}$

$$pc = \left[(50.511 \text{ MeV})^2 - (0.511 \text{ MeV})^2\right]^{\frac{1}{2}} = 50.508 \text{ MeV} = 5.05\times10^7 \text{ eV}$$

$$\lambda = \frac{1240 \text{ eV.nm}}{5.05\times10^7 \text{ eV}} = 2.46\times10^{-5} \text{ nm} = \boxed{2.46\times10^{-14} \text{ m}}$$

(c) I will calculate relativistically. $mc^2 = 938.3 \text{ MeV}$.

$E_{TOT} = 988.3 \text{ MeV}$.

$$pc = \left[(988.3 \text{ MeV})^2 - (938.3 \text{ MeV})^2\right]^{\frac{1}{2}} = 310.4 \text{ MeV}$$

$$\lambda = \frac{1240 \text{ eV.nm}}{310.4 \text{ MeV}} = 4.0\times10^{-6} \text{ nm} \qquad \boxed{\lambda = 4.0\times10^{-15} \text{ m}}$$