29) From class notes $T = \left[1 + \frac{V_0^2}{16E(V_0-E)}\left(e^{\alpha a} - e^{-\alpha a}\right)^2\right]^{-1}$

(a)

$$\alpha a = \left[\frac{2m(V_0-E)}{\hbar^2}\right]^{\frac{1}{2}} \cdot a = \left[\frac{2(5.11\times10^5 eV)(1eV)}{(197.3 eV\cdot nm)^2}\right]^{\frac{1}{2}} \cdot 10nm$$

$$= 51.23$$

$$T = \left[1 + \frac{9^2}{16(2)(1)}\left(e^{51.23} - e^{-51.23}\right)^2\right]^{-1} = \boxed{1.25\times10^{-45}}$$

(b) $$\alpha a = \left[\frac{2(3730\ MeV)(1\ MeV)}{(197.3\ MeV\cdot fm)^2}\right]^{\frac{1}{2}} \cdot 10 fm = 4.38$$

$$T = \left[1 + \frac{9}{16(2)(1)}\left(e^{4.38} - e^{-4.38}\right)^2\right]^{-1} = \boxed{5.61\times10^{-4}}$$

30) First we calculate the tunneling probability. We take $T = e^{-2G}$ where

$$G = \int_a^b \left[\frac{2m(V(x)-E)}{\hbar^2}\right]^{\frac{1}{2}} dx$$

where the integral extends over the classically forbidden region, from R to the point b where $V(b) = E$

$$\Rightarrow \quad \frac{l(l+1)\hbar^2}{2mb^2} = E \Rightarrow b = \left[\frac{l(l+1)\hbar^2}{2mE}\right]^{\frac{1}{2}}$$

$$G = \int_R^b \left[\frac{2m}{\hbar^2}\left(\frac{l(l+1)\hbar^2}{2mx^2} - E\right)\right]^{\frac{1}{2}} dx$$

$$= \int_R^b \left[\frac{l(l+1)}{x^2} - \frac{2mE}{\hbar^2}\right]^{\frac{1}{2}} dx$$

$$= \int_R^b \frac{\sqrt{l(l+1)}}{x}\left[1 - \frac{2mE}{l(l+1)\hbar^2}x^2\right]^{\frac{1}{2}} dx$$

Substitute

$$y = \left[\frac{2mE}{l(l+1)\hbar^2}\right]^{\frac{1}{2}} x \qquad \frac{dx}{x} = \frac{dy}{y}$$

$$G = \sqrt{l(l+1)} \int_{y_1}^{y_2} \frac{1}{y}\left[1 - y^2\right]^{\frac{1}{2}} dy$$

The new limits are $\boxed{y_1 = \left[\frac{2mE}{\ell(\ell+1)\hbar^2}\right]^{\frac{1}{2}} R}$ and

$$y_2 = \left[\frac{2mE}{\ell(\ell+1)\hbar^2}\right]^{\frac{1}{2}} b = \left[\frac{2mE}{\ell(\ell+1)\hbar^2}\right]^{\frac{1}{2}} \left[\frac{\ell(\ell+1)\hbar^2}{2mE}\right]^{\frac{1}{2}} = 1$$

$$G = \sqrt{\ell(\ell+1)} \int_{y_1}^{1} \frac{1}{y}\left[1-y^2\right]^{\frac{1}{2}} dy$$ See integral # 203.

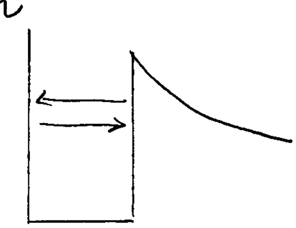
$$G = \sqrt{\ell(\ell+1)} \left[[1-y^2]^{\frac{1}{2}} - \ln \frac{1+[1-y^2]^{\frac{1}{2}}}{y} \right]_{y_1}^{1}$$

$$= \sqrt{\ell(\ell+1)} \left[0 - \ln \tfrac{1}{1} - [1-y_1^2]^{\frac{1}{2}} + \ln \frac{1+[1-y_1^2]^{\frac{1}{2}}}{y_1} \right]$$

$$G = \sqrt{\ell(\ell+1)} \left\{ \ln \frac{1+\sqrt{1-y_1^2}}{y_1} - \sqrt{1-y_1^2} \right\}$$

where y_1 is given above. Calculating $T = e^{-2G}$ gives the probability of escape in 1 try. Then

dP = probability of escape in time dt
= $T \times$ # of tries in time dt = $T \cdot dn$



For each "try" the particle must cross the well twice $\Rightarrow$ t = time interval between tries = $2R/v$. Then the # of tries in dt will be $dt/t = \frac{v}{2R} dt$

$$dP = T \cdot \frac{v}{2R} dt$$ ← average

Roughly speaking, the lifetime would be the time required for dP to reach ~50%

$$0.5 = T \cdot \frac{v}{2R} \cdot t$$

$$t = \frac{R}{T \cdot v}$$

31) This is just a standard square-well problem. To find the energies we need to solve the transcendental equations

$$\tan y = \frac{1}{y}\left[\lambda - y^2\right]^{\frac{1}{2}}$$

2.060

for even states and

.650

$$-\cot y = \frac{1}{y}\left[\lambda - y^2\right]^{\frac{1}{2}}$$

for odd states. Here

$$\lambda \equiv \left[\frac{2mV_0}{\hbar^2}a^2\right] = \frac{2(5.11\times10^5 eV)(1eV)}{(197.3\,eV\cdot nm)^2}(0.5nm)^2$$

$$= 6.562$$

The solutions can be found in various ways. For example write $y = \tan^{-1}\frac{1}{y}\left[\lambda - y^2\right]^{1/2}$ and iterate:

$$y_{n+1} = \tan^{-1}\frac{1}{y_n}\left[\lambda - y_n^2\right]^{\frac{1}{2}}$$

converges quickly. for the lowest state. I find the 1st two states at

$$y = 1.1188 \quad \text{and} \quad y = 2.1474$$

Here

$$y = ka = \left[\frac{2mE}{\hbar^2}\right]^{\frac{1}{2}}a \Rightarrow y^2 = \frac{2mE}{\hbar^2}a^2$$

$$E = \frac{y^2\hbar^2}{2ma^2} = y^2\frac{(197.33eV\cdot nm)^2}{2(5.11\times10^5 eV)(0.5nm)^2}$$

$$\boxed{E_1 = .0.191\,eV} \qquad \boxed{E_2 = 0.703\,eV}$$

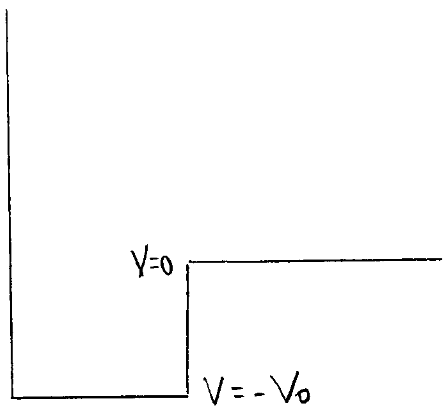
32) For this we only need the value of λ.

$$\lambda = \frac{2mV_0}{\hbar^2} a^2 = \frac{2(5.11\times10^5 eV)(100 eV)}{(197.3 eV.nm)^2}(0.5nm)^2 = 656$$

$$\sqrt{\lambda} = 25.62 = 16.3\frac{\pi}{2}$$

The rule is that when $\sqrt{\lambda}$ is between $n\frac{\pi}{2}$ and $(n+1)\frac{\pi}{2}$ there will be $n+1$ bound states $\Rightarrow$. we will get $\boxed{17}$

33)



We have bound states whenever $E<0$. Inside the well

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi - V_0\psi = E\psi$$

$$\frac{d^2}{dx^2}\psi = -\frac{2m}{\hbar^2}(E+V_0)\psi = -k^2\psi$$

$$\Rightarrow \psi = A\sin kx + B\cos kx$$

We need $\psi\to 0$ as $x\to 0$ so only the $\sin kx$ solution remains.

$$\boxed{\psi_1 = A\sin kx}$$

Outside the well $V=0$ $\Rightarrow$ $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = E\psi$

$$\frac{d^2}{dx^2}\psi = -\frac{2mE}{\hbar^2}\psi$$

Here $E<0$ so we write $\alpha = \sqrt{-2mE/\hbar^2}$ $\quad \frac{d^2}{dx^2}\psi = \alpha^2\psi$

and

$$\boxed{\psi_2 = Ce^{-\alpha x}}$$

The solutions are exactly identical to the odd solutions of the ordinary finite square well. Matching gives

$$A\sin ka = Ce^{-\alpha a}$$

$$kA\cos ka = -\alpha Ce^{-\alpha a}$$

$$\Rightarrow \boxed{k\cot ka = -\alpha}$$

Here $\lambda = \left[\frac{2mV_0}{\hbar^2}\right]a^2 = \frac{2m}{\hbar^2}a^2\left(\frac{32\hbar^2}{ma^2}\right) = 64 \qquad \sqrt{\lambda} = 8$

We expect 1 bound state for $\frac{\pi}{2} < \sqrt{\lambda} < \frac{3\pi}{2}$

2 " " " $\frac{3\pi}{2} < \sqrt{\lambda} < \frac{5\pi}{2}$ etc.

We have $\sqrt{\lambda} = 5.09\ \pi/2$ so there should be 3 bound states.

The 3^{rd} solution is at $y = 7.95732 = ka$

The corresponding value of α is found by writing.

$$(\alpha a)^2 = -\frac{2mE}{\hbar^2}a^2 = \frac{2mV_0}{\hbar^2}a^2 - \frac{2m(E+V_0)}{\hbar^2}a^2 = \lambda - (ka)^2$$

$\Rightarrow$

$$(\alpha a) = \sqrt{64 - y^2} = \underline{0.8253}$$

Normalization integrals. Integrate the $x<a$ and $x>a$ parts separately.

$$N_1 = \int_0^a |A|^2 \sin^2 kx\, dx = |A|^2 \frac{1}{k}\int_0^y \sin^2 u\, du \qquad u = kx$$

$$= \frac{|A|^2}{k}\frac{1}{2}\left(u - \sin u \cos u\right)\Big|_0^y = \frac{|A|^2}{2k}\left[y - \sin y \cos y\right]$$

$$N_2 = \int_a^\infty |C|^2 e^{-2\alpha x}\, dx = \frac{|C|^2}{2\alpha}\left[-e^{-2\alpha x}\right]_a^\infty$$

$$= \frac{|C|^2}{2\alpha} e^{-2\alpha a}$$

Prob of being outside is $\frac{N_2}{N_1+N_2} = \frac{1}{1+N_1/N_2}$

$$\frac{N_1}{N_2} = \left|\frac{A}{C}\right|^2 \frac{\alpha}{k}\left[y - \sin y \cos y\right]e^{2\alpha a}$$

From the matching equations $A \sin ka = Ce^{-\alpha a}$ so

$$\frac{A}{C} = e^{-\alpha a}/\sin y \quad \Rightarrow \quad \frac{N_1}{N_2} = \left(\frac{\alpha}{k}\right)\frac{\cancel{e^{-2\alpha a}}}{\sin^2 y}\left[y - \sin y \cos y\right]\cancel{e^{2\alpha a}}$$

and we also know that $-k\cot ka = \alpha$ so

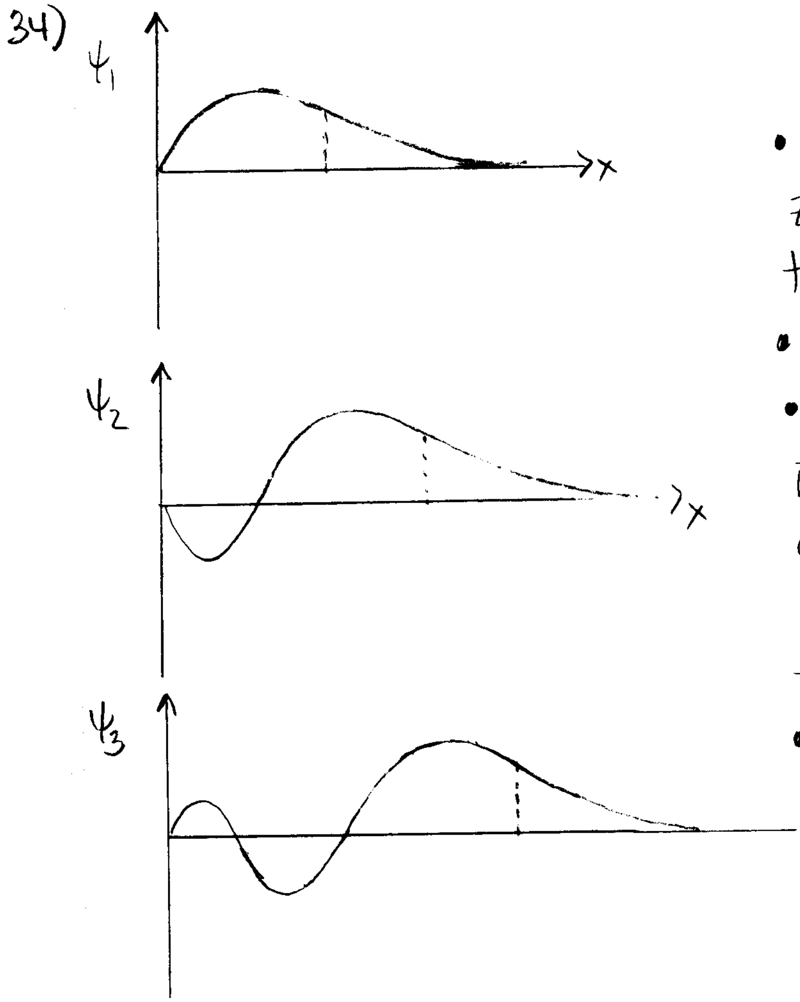
$$\frac{\alpha}{k} = -\cot y = -\frac{\cos y}{\sin y} \qquad \frac{N_1}{N_2} = -\frac{\cos y}{\sin^3 y}\left[y - \sin y \cos y\right]$$

$$= 0.845$$

$$P_{outside} = \frac{1}{1+0.845} = \boxed{0.542} \Rightarrow \boxed{54.2\%}$$

More likely outside than inside

34)



Notes:

- The lowest state has zero nodes, the next 1, the next 2, etc
- $\psi \to 0$ at $x=0$ (since $V \to \infty$)
- The wave function has an inflection point at the classical turning point. As E increases this moves out to larger x.
- Inside the well, the relative curvature of $\psi(x)$ gradually gets smaller as the local kinetic energy $(E-V)$ gets smaller
- Either sign for $\psi(x)$ is OK.