Physics 449 **RELATIVITY PROBLEMS** **Spring 2009**

1) Event A takes place on planet X at time t_0. 50 seconds later a second event, B, takes place on planet Y. The planets are separated by a distance $d = 2.0\times10^{10}$ m.

   (a) Find a frame of reference, S', in which the two events are simultaneous. What is the velocity (magnitude and direction) of S' relative to the frame in which the planets are at rest?

   (b) Is it possible that event A was the cause of event B? Explain.

2) Suppose event A is the cause of event B. If this is the case, it must be possible for information to travel the distance d from A to B in time $t = t_B - t_A$, and since information cannot travel faster than c we have the "causality condition" $d \leq ct$.

   (a) Demonstrate mathematically that if the causality condition holds in one reference frame, it will hold in all frames.

   (b) Show that when the causality condition holds, the time order of the events is the same in all frames of reference.

3) An atom moving at velocity $v = 0.40\,c$ in the $+z$ direction emits an electron. In the rest frame of the atom, the electron's velocity is always $0.80\,c$. Find the velocity (magnitude and direction) of the electron in the lab frame if the emission, in the atom's rest frame is:

   (a) along the $+z$ direction;

   (b) along the $-z$ direction;

   (c) along the $+y$ direction.

   Assume that $m_e \ll m_{\text{atom}}$ so that recoil can be neglected.

4) Starting from the velocity transformation formulas (Eqs. (51-53) of the Relativity Notes), show that $\vec{u}'$ has magnitude c whenever $\vec{u}$ has magnitude c.

5) (a) Show that as $\beta \to 1$, $1 - \beta \to \frac{1}{2}(mc^2/E)^2$.

   (b) Find the energy, E, of electrons with velocity $v = 0.9999c$ and $v = 0.99999c$.

6) (a) A π meson at rest ($m = 140\ \text{MeV/c}^2$) decays into a muon ($m = 105\ \text{MeV/c}^2$) and a neutrino (assume $m = 0$). Use energy and momentum conservation to find the momentum of the neutrino and the kinetic energy of the muon. (For the neutrino use $E = pc$.)

   (b) Repeat the calculation for a π meson moving at $v = 0.5c$, assuming that the neutrino is emitted along the direction of motion. Work the problem by applying the Lorentz transformation to the momentum 4-vectors obtained in part (a). Check that the total energy in the final state is equal to the initial total energy of the π.

7) A particle of unknown mass and velocity decays into two π mesons. One π is moving at speed 0.9c in the $+z$ direction and the other at speed 0.4c in the $-z$ direction. Find the mass of the unknown particle. What was it's original velocity?

8) Suppose that in the lab we produce an electric field $\mathcal{E}_0$ in the z-direction and a magnetic field $\mathcal{B}_0$ in the x-direction. Find the electric and magnetic field components in a frame of reference S$'$ which is moving at velocity β in the z-direction.

9) Hydrogen atoms at rest emit light at a wavelength $\lambda = 656\,\text{nm}$. Find the wavelength of the light emitted in the forward direction and in the backward direction by hydrogen atoms moving at $v = 0.3\,c$. Work the problem by treating the light as a collection of photons of energy $E = h\nu = hc/\lambda$ and momentum $p = E/c$, and applying the Lorentz transformation to the momentum 4-vector. Your

answers should agree with the results obtained from the ordinary relativistic doppler shift formula, which is normally derived from the wave picture of light using time dilation arguments.

10) Electrons of energy E bombard a target of electrons at rest. Find the threshold energy for production of an e^+ e^- pair in the collision.