PRACTICE PROBLEMS (from old exams)

1) Find the mean lifetime for spontaneous emission for a particle of mass m and charge e in the first excited state of a 3-dimensional harmonic oscillator well. The excited state has energy $\frac{5}{2}\hbar\omega$ and angular momentum $l=1$, while the ground state has $E=\frac{3}{2}\hbar\omega$ and $l=0$. The radial wave functions are

$$R(r) = 2\left[\frac{\alpha^3}{\pi}\right]^{\frac{1}{4}} e^{-\alpha r^2/2}$$

for the ground state and

$$R(r) = 2\left[\frac{\alpha^3}{\pi}\right]^{\frac{1}{4}} \sqrt{\frac{3\alpha}{2}}\; r\; e^{-\alpha r^2/2}$$

for the excited state, where $\alpha = \sqrt{km/\hbar^2}$. The three degenerate excited states have equal lifetimes, so you only need to do the calculation for one case. The relevant spherical harmonic functions are

$$Y_0^0 = \sqrt{\frac{1}{4\pi}} \qquad Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta e^{i\phi} \qquad Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta \qquad Y_1^{-1} = \sqrt{\frac{3}{8\pi}}\sin\theta e^{-i\phi}.$$

Do whatever integrals you can, or at least reduce them to some simple form. It helps to make use of the fact that the radial wave functions given above are normalized. Remember

$$B = \frac{4}{3}\frac{e^2}{4\pi\epsilon_0}\frac{\omega_{mn}^3}{\hbar c^3}\left[\ |x_{mn}|^2 + |y_{mn}|^2 + |z_{mn}|^2\ \right].$$

2) The effective potential of the Na^+Cl^- molecule can be approximated by

$$V(R) = V_0 - \frac{e^2}{4\pi\epsilon_0}\frac{1}{R} + \frac{A}{R^3}$$

where $A = 0.03\ \text{eV}\cdot\text{nm}^3$.

(a) Find the equilibrium separation R_0.

(b) Determine the excitation energy of the first ($k=1$) rotational state. The reduced mass of the system is $1.33\times10^{10}\ \text{eV}/c^2$.

(c) Estimate the energy spacing of the vibrational levels, $\hbar\omega$.

3) Photons are absorbed by hydrogen atoms in the $1s$ ground state leading to excitation of the $2p$ state. Which m values are populated in the following situations:

(a) the photons are incident along the z-axis and are linearly polarized along $\hat{x}$;

(b) the photons are incident along the x-axis and are linearly polarized along $\hat{z}$;

(c) the photons are incident along the z-axis and are right circularly polarized.

4) Consider a simple two-level system in thermal equilibrium at temperature T. Suppose there are 10^5 degenerate states at each energy level and that the energy spacing is $\epsilon = 1.61\,\text{kT}$ (so that $e^{\epsilon/\text{kT}} = 5$). Find the number of electrons in each state if the total number of electrons is 1.6×10^5.