# HOMEWORK 5 SOLUTIONS

20) (a) There are 10 single-electron states so $\boxed{N = \frac{10 \cdot 9}{2} = 45}$

We can have $S=0$ (antisymmetric) or $S=1$ (symmetric). The allowed $l$ values are $l = 4,3,2,1,0$. Here $l = 4,2,0$ will be symmetric and $l = 3,1$ will be antisymmetric. To get an antisymmetric wave function the choices are ${}^1G$, ${}^3F$, ${}^1D$, ${}^3P$ and ${}^1S$. Now $\vec{J} = \vec{L} + \vec{S}$ gives

$$\boxed{{}^1G_4,\ {}^3F_2,\ {}^3F_3,\ {}^3F_4,\ {}^1D_2,\ {}^3P_0,\ {}^3P_1,\ {}^3P_2,\ {}^1S_0}$$

Total $9+5+7+9+5+1+3+5+1 = 45$

(b) There are 10 choices for the 3d electron and 10 for the 4d electron $\Rightarrow$ $\boxed{N = 100}$

Here we get all possible $l, s$ combinations ${}^1G$, ${}^3G$, ${}^1F$, ${}^3F$ etc

$$\Rightarrow \boxed{{}^1G_4\ {}^3G_3\ {}^3G_4\ {}^3G_5\ {}^1F_3\ {}^3F_2\ {}^3F_3\ {}^3F_4\ {}^1D_2\ {}^3D_1\ {}^3D_2\ {}^3D_3\ {}^1P_1,\ {}^3P_0,\ {}^3P_1,\ {}^3P_2,\ {}^1S_0,\ {}^3S_1}$$

Total $9+7+9+11+7+5+7+9+5+3+5+7+3+1+3+5+1+3 = 100$

(c) 6 choices for 4p 14 choices for 4F $\Rightarrow \boxed{84}$

$s = 0$ or $1$ $l = 2,3,4$ all combinations allowed.

$$\boxed{{}^1G_4\ {}^3G_3\ {}^3G_4\ {}^3G_5\ {}^1F_3\ {}^3F_2\ {}^3F_3\ {}^3F_4\ {}^1D_2\ {}^3D_1\ {}^3D_2\ {}^3D_3}$$

$9+7+9+11+7+5+7+9+5+3+5+7 = 84$

(d) For $(2p)^2$ there are 15 possibilities and for $(3p)^1$ there are 6 so we should get $\boxed{N = 15 \times 6 = 90}$

For the $(2p)^2$ electrons we 3 possible $l, s$ combinations,

i) $l=0, s=0$ ii) $l=1, s=1$ iii) $l=2, s=0$ where the quantum numbers are for $\vec{L_1}+\vec{L_2}$ and $\vec{S_1}+\vec{S_2}$. So now we add $l_3=1$ and $s_3=\frac{1}{2}$

i) $l=0$, $l_3=1$ $\Rightarrow$ $l_{TOT}=1$

$s=0$ $s_3=\frac{1}{2}$ $\Rightarrow$ $s_{TOT}=\frac{1}{2}$ $\Rightarrow$ $\boxed{{}^2P_{\frac{1}{2}},\ {}^2P_{\frac{3}{2}}}$

ii) $l=1$, $l_3=1$ $\Rightarrow$ $l_{TOT}=0,1,2$

$s=1$ $s_3=\frac{1}{2}$ $\Rightarrow$ $S_{TOT}=\frac{1}{2},\frac{3}{2}$

$\Rightarrow$ $\boxed{{}^2S_{\frac{1}{2}},\ {}^4S_{\frac{3}{2}}},\ \boxed{{}^2P_{\frac{1}{2}},\ {}^2P_{\frac{3}{2}},\ {}^4P_{\frac{1}{2}},\ {}^4P_{\frac{3}{2}},\ {}^4P_{\frac{5}{2}},\ {}^2D_{\frac{3}{2}},\ {}^2D_{\frac{5}{2}},\ {}^4D_{\frac{1}{2}}\ \ {}^4D_{\frac{3}{2}}\ \ {}^4D_{\frac{5}{2}}\ \ {}^4D_{\frac{7}{2}}}$

iii) $l=2$, $l_3=1$ $\Rightarrow$ $l_{TOT}=1,2,3$

$S=0$, $S_3=\frac{1}{2}$ $\Rightarrow$ $S_{TOT}=\frac{1}{2}$ $\Rightarrow$ $\boxed{{}^2P_{\frac{1}{2}},\ {}^2P_{\frac{3}{2}},\ {}^2D_{\frac{3}{2}},\ {}^2D_{\frac{5}{2}},\ {}^2F_{\frac{5}{2}},\ {}^2F_{\frac{7}{2}}}$

Total states $= 2+4+2+4+2+4+2+4+6+4+6+2+4+6+8$
$+2+4+4+6+6+8 = 90$ ✓

21) The states are ${}^1G_4$, ${}^3F_2$, ${}^3F_3$, ${}^3F_4$, ${}^1D_2$, ${}^3P_0$, ${}^3P_1$, ${}^3P_2$, ${}^1S_0$

Rule # 1 : Higher $S$ $\Rightarrow$ lower energy $\Rightarrow$ triplets lower than singlets.

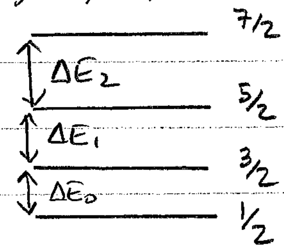
Rule # 2 Higher $l$ $\Rightarrow$ lower energy $\Rightarrow$ F below P
G below D below S.

Rule # 3 Lower $j$ $\Rightarrow$ lower energy.

Lowest to highest:

${}^3F_2$, ${}^3F_3$, ${}^3F_4$ ; ${}^3P_0$, ${}^3P_1$, ${}^3P_2$ ; ${}^1G_4$, ${}^1D_2$, ${}^1S_0$

22) For a $^4D$ state $\quad l=2 \quad s=\frac{3}{2} \quad \Rightarrow \quad j=\frac{7}{2},\frac{5}{2},\frac{3}{2},\frac{1}{2}$




According to the interval rule the splitting between 2 states is proportional to the $j$ of the higher one

$$\Delta E\left(\tfrac{7}{2}-\tfrac{5}{2}\right) = \tfrac{7}{2}C$$
$$\Delta E\left(\tfrac{5}{2}-\tfrac{3}{2}\right) = \tfrac{5}{2}C$$
$$\Delta E\left(\tfrac{3}{2}-\tfrac{1}{2}\right) = \tfrac{3}{2}C$$

From the picture with my definitions for the $\Delta E$'s

| | |
|---|---|
| $\boxed{\Delta E_1/\Delta E_0 = 5/3}$ | $\boxed{\Delta E_2/\Delta E_0 = 7/3}$ |

23) $^4F_{3/2} \Rightarrow \quad l=3 \quad s=\frac{3}{2} \quad j=\frac{3}{2}$

$$g = 1+\frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)} = 1+\frac{\frac{3}{2}\left(\frac{5}{2}\right)+\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)-3(4)}{2\,\frac{3}{2}\cdot\frac{5}{2}}$$

$$= 1+\frac{15+15-48}{30} = 1-\frac{18}{30} = 1-\frac{3}{5} = \boxed{\frac{2}{5}}$$

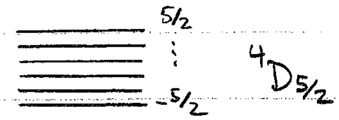
$^4D_{\frac{5}{2}} \quad l=2 \quad s=\frac{3}{2} \quad j=\frac{5}{2} \quad \Rightarrow \quad g = 1+\frac{\frac{5}{2}\left(\frac{7}{2}\right)+\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)-2(3)}{2\left(\frac{5}{2}\right)\left(\frac{7}{2}\right)}$

$$= 1+\frac{35+15-24}{70} = 1+\frac{26}{70} = \boxed{\frac{48}{35}}$$

The energy shifts are given $E^{(1)} = g\mu_B B m_j$ so we get

$^4D_{\frac{5}{2}} \quad g\mu_B B = \left(\frac{48}{35}\right)(5.788\times10^{-5}\,eV/T)(0.8T) = \boxed{6.35\times10^{-5}\,eV}$

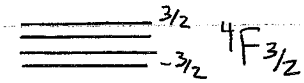
$^4F_{\frac{3}{2}} \quad$ " $= \left(\frac{2}{5}\right)$ " " $= \boxed{1.85\times10^{-5}\,eV}$




For the emission we have

$$E_\gamma = E_\gamma(B=0) + (m_i g_i - m_f g_f)\mu_B B$$

The selection rule is $\Delta m = 0, \pm 1$ so for each $^4F_{3/2}$ final state there are 3 possible $^4G_{5/2}$ initial states $\Rightarrow$ 12 transitions in all.




Given $E_\gamma$ we calculate the wavelength by $E = hc/\lambda$

$\Rightarrow$

$\lambda = \frac{hc}{E}$ All the energies are close to $E_0$ $(E = E_0 + \delta E)$

and so the shift in wavelength is

$$\delta\lambda \simeq \frac{d\lambda}{dE}\delta E = -\frac{hc}{E^2}\delta E = -\frac{hc}{E_0}\frac{\delta E}{E_0} = -\lambda_0 \frac{\delta E}{E_0}$$

$$\underline{\lambda =} \lambda_0 + \delta\lambda \simeq \underline{\lambda_0(1 - \delta E/E_0)} \quad \text{where } \lambda_0 = 375\,\text{nm}$$

and $E_0 = hc/\lambda_0 = 1240\,\text{eV}\cdot\text{nm} / 375\,\text{nm} = 3.307\,\text{eV}$.

| $m_i$ | $m_f$ | $\delta E$ | $\lambda$ |
|---|---|---|---|
| 5/2 | 3/2 | $1.31 \times 10^{-4}$ eV | 374.9852 |
| 3/2 | 3/2 | 0.67 " | 374.9924 |
| 1/2 | 3/2 | 0.04 " | 374.9996 |
| 3/2 | 1/2 | 0.86 " | 374.9903 |
| 1/2 | 1/2 | 0.22 " | 374.9975 |
| -1/2 | 1/2 | -0.41 " | 375.0046 |
| 1/2 | -1/2 | 0.41 " | 374.9954 |
| -1/2 | -1/2 | -0.22 " | 375.0025 |
| -3/2 | -1/2 | -0.86 " | 375.0097 |
| -1/2 | -3/2 | -0.04 " | 375.0004 |
| -3/2 | -3/2 | -0.67 " | 375.0076 |
| -5/2 | -3/2 | -1.31 " | 375.0141 |